A Dynamic Dual-Response-Surface Methodology for Optimal Design of a Permanent-magnet Motor Using Finite-element Method

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This paper proposes a novel strategy to accelerate the optimization process for designing a permanent-magnet (PM) motor. In order to compute an objective function, a finite-element method (FEM) with the assistance of a dynamic dual-response-surface model (dual-RSM) is presented. The FEM serves as a basic tool to compute the objective function and at the same time, its results are used to train the dual-RSM model. A radial basis function (RBF) and a moving least square (MLS) approximation are both employed to build up the dual-RSM model. The results which are obtained from the dual-RSM model are dynamically compared together with the results from the FEM, which determines whether the accuracies of the dual-RSM are high enough to replace the FEM computation. This strategy makes the FEM computation not always required at every sample point in the optimization process while ensures the accuracies of RSM. The optimal design of a permanent-magnet (PM) synchronous motor is taken as an example to demonstrate the effectiveness of the proposed optimization methodology. Comparing with traditional optimal method which only uses FEM for the evaluation of the objective function, the proposed method can dramatically reduce the computing time.

***Keywords*— Electric motor, finite-element method, moving least square approximation, optimization method, particle swarm optimization, radial basis function, response surface model.**

I. Introduction

To approach the best design of electric machines, many optimal design algorithms including deterministic and non-deterministic algorithms have been developed and applied [1]. As finite-element method (FEM) in time domain can precisely simulate the operation of electric machines, it has been employed to compute the objective functions in optimal design of the machines. As transient FEM requires meshing and numerical computation at each time step [2-3], the precise FEM approximation of the electromagnetic field inside the machines, especially in the process of optimization, often requires huge computing time. Therefore, it is highly expected to evaluate the objective functions using a fast approximate process to considerably reduce the computing time of FEM analysis [4]. Recently, some researchers have applied a response surface model (RSM) to the optimization of the motor design and torque control [5-6]. The RSM is normally introduced to reduce the number of FEM computations, then greatly reduce the cost of compution [7]. Normally the basic optimization involves only one type of RSM. However, the accuracy of the single RSM may not be as high as it is expected; and the error from the RSM may also not be easily estimated. If the results from the RSM is not accurate, the optimization will become meaningless.

In this paper a novel strategy to compute the objective functions using FEM and a dynamic dual-RSM is presented. The design of a permanent magnet (PM) motor for its high performance is taken as the example to demonstrate the proposed method. The advantage of the proposed method is that the overall computing time for evaluating the objective functions can be significantly reduced while the accuracies of the solutions are still ensured.

II. Dual-response-surface Model

The proposed dual-RSM contains two types of RSMs. The first type of RSM employed is the multi quadric (MQ) radial basis function which is widely being used as global interpolation and is very impressive in the viewpoint of smoothness and fitting ability with limited sampling points [5-6, 8]. Another type of the RSM used in this paper is the moving least square approximation (MLS) which is proved successful as an effective and stable global technique for local fitting [9]. The particle swarm optimization (PSO) method is used for selecting sample points, which is one of the widely used optimization techniques for PM motor design [10].

During optimization process, both the RBF model and MLS model are built based on a collection of sample points from FEM simulations. A pre-set error is one of the standards for choosing proper model for the optimization. Both types of models will be dynamically updated when more sample points from FEM simulations are available. If the difference between the results from the two RSM models is smaller than the pre-set error, it means that the results from RSMs may be acceptable. The errors of the RSMs on the point (from the list of the FEM simulated points) which is the nearest to the current point are further checked. If all of the errors are small, the FEM simulation on this point is not required. Otherwise, the FEM computation is needed and this point will be added to the collection of the sample points for training RSMs further. The block diagram of the evaluation process of the objective function is shown in Fig. 1.

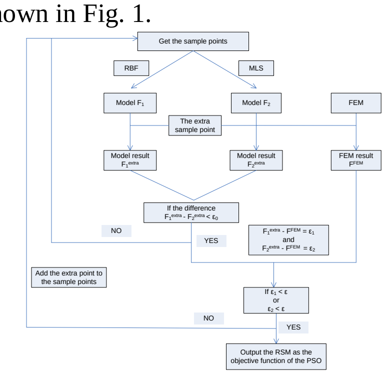

Fig. 1. Block diagram of the evaluation process of objective function

III. General Formulations

The aim of building RSMs is to obtain an approximate model of the object function for the motor design. The RSMs are trained by the results from the FEM, and their accuracies are evaluated by the difference of the results of the two models and the comparison to the FEM results.

The objective function of the optimization for the PM motor is the power efficiency η. The output power is computed by $P_2 = T \cdot \omega$. The coreloss is computed acording to the magnetic field distribution.

A. Optimization Algorithm

The basic PSO algorithm is:

$$v_d^i(k+1) = \omega v_d^i(k) + c_1 r_1 \left(p_d^i - x_d^i(k)\right) + c_2 r_2 \left(g_d^i - x_d^i(k)\right), \quad (1)$$

$$v_d^i(k+1) = v_d^i(k+1) \cdot v_d^{max} / |v_d^i(k+1)| \quad (if\ |v_d^i(k+1)| > v_d^i), \quad (2)$$

$$x_d^i(k+1) = x_d^i(k) + v_d^i(k+1). \quad (3)$$

where $\boldsymbol{v}_i(k) = \{v_d^i\}$ and $\boldsymbol{x}_i(k) = \{x_d^i\}$ are the velocity vector and the position of the dth variable on the ith sample point, respectively; $\boldsymbol{g}_i = \{g_d^i\}$ is the best point among the neighboring to the ith sample point; v_d^{max} is the maximum value of velocity of the dth variable. The velocity and the direction of the ith sample point at the next step is determined by the fomer fomulas.

B. RSM Formulations

In the design variable space, the RSM of the MQ-RBF is constructed as follows:

$$f(x) = \sum_{i=1}^{N} c_i H_i(r), \quad (4)$$

in which c_i is the coefficient to the ith term associated with the sample point x_i, and the interpolation function is

$$H(r) = (r^2 + h)^{\beta}, \ (\beta = 0.5), \quad (5)$$

where $r = ||x - x_i||$ is the Euclidean direction between x and x_i; and h is the shape parameter.

The RSM of the MLS is constructed as follows:

$$f = \sum_{i=1}^{N} a_i(x) b_i(x), \quad (6)$$

where $\boldsymbol{b} = \{b_i(x)\}, i = 1,2,\dots,n (n < N)$ is the basis of a complete polynomial with order N. The unknown $a_i(x)$ satisfies the equation

$$a_i(x) = \boldsymbol{A}(x)^{-1} \boldsymbol{B}(x) \boldsymbol{U}. \quad (7)$$

The MLS approximation of the object function is given by

$$f^h = \sum_{i=1}^{N} \Phi_i(x) u_i = \boldsymbol{\Phi}(x) \boldsymbol{U}, \quad (8)$$

in which u_i is the node value, and $\boldsymbol{U} = \{u_1, u_2, \dots, u_N\}^{\mathrm{T}}$. The $\boldsymbol{\Phi}(x)$ is the shape function of MLS approximation. It is essential to choose a proper shape function for the MLS.

The pre-set error is an important index in controlling the accuracies of the RSMs. A proper pre-set error can help to reduce the computing time and obtain a satisfactory result.

IV. Numerical Experiment

To validate the proposed method mentioned above, a PM synchronous motor is designed. The design variables of the studied motor for the numerical experiment are given in Table I. The power efficiency of the motor versus the computing time during the optimization process is shown in Fig. 2. More detained results will be given in the full paper.

TABLE I
Design variables of the studied PM motor

Design Parameters	Value ranges
h_1 (thickness of the outer ring)	22-26mm
h_2 (thickness of the permanent magnets)	8-14mm
b_t (tooth width)	14-18mm
h_4 (slot depth)	40-45mm

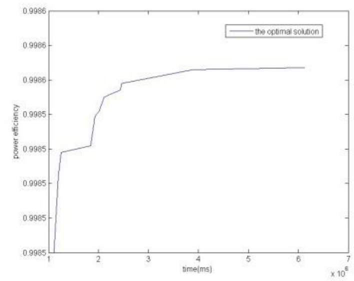

Fig. 2. The power efficiency of the motor versus computing time during optimization.

V. Conclusion

The proposed strategy employs a dual-RSM methodology to dynamically reestablish the objection function according to the results of FEM simulations. The numerical example of the optimal design of a PM motor shows that the developed computer program based on the proposed algorithm can significantly reduce the number of times of FEM simulations and accelerate the optimization process while the accuracies of the dual-RSM is ensured.

VI. References

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